Next, the form of the invariant multiplier is investigated, and it is shown that the eliminant of $C_{m,m}$ and $C_{p,m+1}$ is a perfect mth power, of which I_{p+q-k} is the mth root. Lastly, the special cases of the quintic and septimic are discussed. The first case is treated exhaustively; for the second, on the other hand, only a summary of the main results obtained, without proofs, is given.

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ON THE GROUPS GENERATED BY TWO OPERATORS SATISFYING THE CONDITION $s_1s_2 = s_2^{-2}s_1^{-2}$.

BY PROFESSOR G. A. MILLER.

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§ 1. *Introduction.*

REMARKABLE general properties may easily be proved as regards the system of groups generated by two operators s_1, s_2 which satisfy the condition expressed by one of the following pair of equivalent equations:

$$s_1s_2 = s_2^{-2}s_1^{-2}, \quad s_1^2s_2^2 = s_2^{-1}s_1^{-1}.$$

From the facts that $(s_2s_1)^2 = s_2s_1s_2s_1 = s_2^{-1}s_1^{-1} = (s_1s_2)^{-1}$ and that s_1s_2 is of the same order as s_2s_1, it results that the order of s_1s_2 is an odd number. From the same equations it follows that s_1s_2 is transformed into a power of itself both by s_1 and by s_2, and hence the cyclic group generated by s_1s_2 is invariant under the group G generated by s_1, s_2. As G is generated both by s_1s_2, s_1 and also by s_1s_2, s_2 it results from the preceding sentence that if s_1^{α} is the lowest power of s_1 which occurs in the cyclic group generated by s_1s_2 it is necessary that s_2^{α} is the lowest power of s_2 occurring in this group and vice versa. Moreover, if s_1 is commutative with $(s_1s_2)^n$ the following equations are satisfied:

$$(s_2s_1)^n = (s_1s_2)^n = (s_2s_1)^{-2n} \quad \text{since} \quad (s_2s_1)^2 = s_2^{-1}s_1^{-1} = (s_1s_2)^{-1}.$$