The maximum value of m is $l^2 - l + 1$,* and the largest value of l is the largest integer in $\frac{1}{2}n$. When n is even, say $2l$, these curves are already included among those treated before for $r = \frac{1}{2}n - 1$. By taking smaller values for l, curves of all the higher genera can be constructed.

3. When n is odd, say $n = 2l + 1$, the lowest genus obtainable directly is $l^2 - 1$, while the highest one from the space curve is $l^2 - l$. By passing a pencil of c_{l+1} through the l^2 base points of a pencil of c_l, the lower genus can be obtained, and the higher one by projection. To obtain the intermediate cases, pass two c_l through $l - 1$ points on a fixed straight line c_1. Through κ of the $l^2 - l + 1$ remaining points of intersection, the same $l - 1$ points on c_1, and two other points on c_1 pass a pencil of c_{l+2}. The two pencils are now to be made projective in such a way that corresponding curves intersect on c_1. The locus of the point of intersection will be a c_{2l+2}, but c_1 will be a factor. The remaining c_{2l+1} will have $\kappa + l - 1$ double points, and no other singularities. κ can have any value from 0 to $l^2 - l + 1$. This completes the solution of the problem.†

CORNELL UNIVERSITY,
July, 1908.

ON PERIODIC LINEAR SUBSTITUTIONS WHOSE COEFFICIENTS ARE INTEGERS.

BY DR. ARTHUR RANUM.

(Read before the American Mathematical Society, September 6, 1907.)

1. THE object of this note is to call attention to one or two theorems that follow easily from the results of my paper in this BULLETIN, April, 1907, volume 13, pages 336–345, by taking into account a theorem of Minkowski's given in *Crelle*,

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*C. Kupper: "Ueber das Vorkommen von linearen Schaaren · · · " *Sitzungsbericht der Böhm. Gesell.*, Prag, 1892, pp. 264–262. In my article "On curves having a net of minimum adjoint curves," BULLETIN, vol. 14, page 70 (1907) I showed how such a net of curves can be actually constructed by rational operations.

† Since the basis points are not independent, Cayley's theorems regarding the configuration of residual points of intersection do not apply. See Küpper, "Bestimmung der Maximalbasis B für eine irreducible μ-fache Mannigfaltigkeit von Curven $n$ter Ordnung," *Monatshefte für Math. u. Physik*, vol. 6, (1895), pp. 5–11, and my own paper, "On birational transformations of curves of high genus," *Amer. Jour. Math.* vol. 30 (1908), pp. 10–20.