

It is of interest to notice that the type III presented itself first in connection with the general transformation theory* of the lineal elements of space. Such a transformation is defined by setting $x_1, y_1, z_1, y'_1, z'_1$ equal to arbitrary functions of x, y, z, y', z' . Lie showed that the only case in which every union is turned into a union is the extended point transformation. There may, however, be other transformations which convert some unions into unions. Examples of this sort have been given by Lie in which the unions considered are the curves of a minimal complex or some other Monge equation (II). The general result is as follows:

The most extensive systems of curves that can be converted into curves by element transformations which are not merely extended point transformations are those defined by differential equations of type III, i. e., precisely those characterized by the Meusnier property.

For any system of type III transformations of this sort may be found. If a system not satisfying an equation III is transformed into curves, the element transformation is necessarily an extended point transformation.

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ON THE DISTANCE FROM A POINT TO A SURFACE.†

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IN a recent number of the BULLETIN,‡ Professor Hedrick has called attention to the fact that the normal distance from a given point to a surface may be a minimum among the distances in every normal plane section without however being a minimum among the distances to the surface. In this connection it may be of interest to observe that for surfaces of a very general type the phenomenon in question can occur only when the given point is a principal center of curvature. In fact, if the equation

* Cf. abstract of the author's paper on this subject in the BULLETIN, vol. 10 (1904), p. 492.

† The theorems established in a previous note (BULLETIN, vol. 13, p. 447) are not new, as I then believed; they are to be found in section 60 of Gourzat's Cours d'Analyse. I take this opportunity of acknowledging Professor Hedrick's kindness in calling my attention to the fact.

‡ BULLETIN, April, 1908, p. 321.