THE INVERSE OF MEUSNIER'S THEOREM.

BY PROFESSOR EDWARD KASNER.

(Read before the American Mathematical Society, April 25, 1908.)

MEUSNIER'S theorem, relating to the curves drawn on an arbitrary surface

(I) $$f(x, y, z) = 0,$$

was extended by Lie* to the curves satisfying any Monge equation of the first order

(II) $$f(x, y, z, y', z') = 0,$$

where primes denote differentiation with respect to x. In this note we show that the theorem is valid in the more general case of curves defined by any equation of the form

(III) $$Ay'' + Bz'' + C = 0,$$

where A, B, C are arbitrary functions of $x, y, z, y' z'$; and that no further extension is possible.

Our problem is to find the most general system of space curves with the *Meusnier property*. This deals with the curvature of the curves of the system which pass through a common point O, in a common direction, and may be stated in any one of the following equivalent ways:

1. The radius of curvature varies as the sine of the angle between the osculating plane and a fixed plane.
2. The circles of curvature generate a sphere.
3. The locus of the centers of curvature is a circle through the point O.
4. The locus of the inverse centers of curvature is a straight line.

It will be convenient to use the last statement, sometimes referred to as Hachette's theorem. Taking the origin at the given point O, and the axis of X along the given direction, we find for the center of curvature

* *Leipziger Berichte*, vol. 50 (1898), p. 1; *Math. Annalen*, vol. 59 (1904), p. 299.