CRITERIA FOR THE IRREDUCIBILITY OF A RECIPROCAL EQUATION.

BY PROFESSOR L. E. DICKSON.

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1. A reciprocal equation $f(x) = x^m + \cdots = 0$ is one for which

$$x^m f(1/x) \equiv cf(x).$$

Replacing x by $1/x$, we see that $f \equiv c^2 f$, $c = \pm 1$. Now $f(x)$ has the factor $x \pm 1$ and hence is reducible, unless m is even and $c = +1$. Further discussion may therefore be limited to equations

$$(1)\quad F(x) \equiv x^{2n} + c_1 x^{2n-1} + c_2 x^{2n-2} + \cdots + c_2 x^2 + c_1 x + 1 = 0$$

of even degree and having

$$(2)\qquad x^{2n} F(1/x) \equiv F(x).$$

Let R be a domain of rationality containing the c's.

Under the substitution

$$(3)\qquad x + 1/x = y,$$

$x^{-n} F(x)$ becomes a polynomial in y,

$$(4)\qquad \phi(y) = y^n + k_1 y^{n-1} + \cdots + k_n,$$

with coefficients in R. By a suitable choice of the c's, the k's may be made equal to any assigned values.

We shall establish in §§ 2–7 the following:

THEOREM. *Necessary and sufficient conditions for the irreducibility of $F(x)$ in the domain R are*

(I) *$\phi(y)$ must be irreducible in R.*

(II) *$F(x)$ must not equal a product of two distinct irreducible functions of degree n.*

The second condition is discussed in §§ 8–10.

2. The irreducibility of $F(x)$ in R implies that of $\phi(y)$. For, if