NOTE ON A CERTAIN EQUATION INVOLVING THE FUNCTION $E(x)$.

BY PROFESSOR R. D. CARMICHAEL.

RECENTLY J. V. Pexider has studied the equation*

$$(1) \qquad E\left(\frac{n+\alpha}{x}\right) - E\left(\frac{n+\alpha}{x+1}\right) = d,$$

where $E(s)$ is the greatest integer $\leqq s$, and where α is zero or a positive quantity less than 1, n is a known positive integer, d is zero or a known positive integer, and x is an unknown positive integer. M. Pexider confines himself chiefly to the case in which $d = 0$ and x is less than n. He finds the values of x which satisfy the equation subject to these restrictions.

In the present note it is proposed to exhibit a simple working method by which the roots can be found in any case. When $d > 0$, x is always less than n, except that x may equal n when $d = 1$. In what follows x is taken always less than n.

If $n + \alpha$ is divided by an integer i, giving the quotient $q + \beta$ where β is zero or a positive quantity less than 1; then if n is also divided by i, the quotient will evidently be $q + y$ where y is zero or a positive quantity less than 1. Hence

$$E\left(\frac{n+\alpha}{i}\right) = E\left(\frac{n}{i}\right).$$

Therefore, the equation

$$(2) \qquad E\left(\frac{n}{x}\right) - E\left(\frac{n}{x+1}\right) = d$$

has the same roots as (1). We may then confine ourselves to the solution of the latter equation as being somewhat the simpler of the two.

Represent n in the form

$$(3) \qquad n = ax + c \qquad (c < x, \ a \neq 0).$$

* *Rendiconti del Circolo Matem. di Palermo*, vol. 24, no. 1, pp. 46–64. For convenience I write the equation in a form somewhat different from that of M. Pexider.