9. Professor Bowden gave an elementary proof by mathematical induction of the formula

$$C_r^{m+n} = \sum_{k=1}^{k=r+1} C_{r-k+1}^m C_{k-1}^n.$$

F. N. Cole,
Secretary.

ON TRIPLE ALGEBRAS AND TERNARY CUBIC FORMS.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, October 26, 1907.)

1. For any field F in which there is an irreducible cubic equation $f(\rho) = 0$, the norm of $x + y\rho + z\rho^2$ is a ternary cubic form C which vanishes for no set of values x, y, z in F, other than $x = y = z = 0$. The conditions under which the general ternary form has the last property are here determined for the case of finite fields. One formulation of the result is as follows:

Theorem. *The necessary and sufficient conditions that a ternary cubic form C shall vanish for no set of values x, y, z in the $GF[p^n]$, $p > 2$, other than $x = y = z = 0$, are that its Hessian shall equal mC, where m is a constant different from zero, and that the binary form obtained from C by setting $z = 0$ shall be irreducible in the field.*

Although I have not hitherto published a proof of this theorem, I have applied it to effect a determination * of all finite triple linear algebras in which multiplication is commutative and distributive, but not necessarily associative, while division is always uniquely possible. I shall here (§ 11) determine these algebras by applying directly the more fundamental conditions from which the preceding theorem is derived.

These ternary cubic forms arise in various other problems; for instance, in the normalization of families of ternary quadratic forms containing three linearly independent forms.

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* *Amer. Math. Monthly*, vol. 13 (1906), pp. 201–205. References are there given to my earlier papers on the subject.