THIRD REPORT ON RECENT PROGRESS IN THE THEORY OF GROUPS OF FINITE ORDER.

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§ 3. Group of Isomorphisms.

In view of the fundamental importance of the group of isomorphisms of any group it is desirable to have theorems by means of which the group of isomorphisms I of a given substitution group G can be readily determined. If any G of degree n contains n distinct subgroups of degree $n - 1$ which are composed of all the substitutions omitting fixed letters, then its substitutions transform these n subgroups just as they transform their own letters and hence G contains no substitution besides identity which is commutative with each one of these n subgroups of degree $n - 1$. When G is transitive it will have n such subgroups, provided it has one subgroup of degree $n - 1$. Any operator which transforms each of these n subgroups into itself must therefore be commutative with every operator of G. If G does not involve a subgroup of degree n which may correspond to such a subgroup of degree $n - 1$ in a holomorphism of G, then its I can be represented as a substitution group of degree n which involves G as an invariant subgroup.

If G is a transitive group and contains a subgroup of degree $n - 1$, its I may be represented as a transitive substitution group whose elements are the subgroups which may correspond to one of the largest subgroups of degree $n - 1$ in a holomorphism of G. If the degree of this transitive group exceeds n, it must be imprimitive and the m conjugate largest subgroups of degree $n - 1$ constitute one system of imprimitivity, while its other systems correspond to subgroups of degree and of index n under G. While these general theorems are frequently directly useful to determine the I of a given G, a number of recent more special theorems find wide application. Among these are the following:

If an abelian group G which involves operators whose orders exceed 2 is extended by means of an operator of order 2 which transforms each operator of G into its inverse, then the I of