condition that the integral $\int_1^2 (\log(-1/y_1') - 1)\, dx$ take an extreme value is

$$y_1'' = 0, \tag{5}$$

the substitution (3) is a member of the defined type and $F_{y'y'} \div F_{yy'} \equiv y' = \text{const.}$ is an integral of (2). $y' = c$, $\pi = \partial F / \partial y' = - y'$ gives $\pi = - c$ for an integral of (1).

CORNELL UNIVERSITY,
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THE MAXIMUM VALUE OF A DETERMINANT.

BY DR. F. R. SHARPE.

HADAMARD* has shown that the maximum value of a determinant when the absolute value of each element does not exceed 1 is $n^{\frac{1}{2}n}$. The square of such a maximum determinant is a determinant having all its elements 0 except those of the principal diagonal. If the elements are restricted to real values, they are each ± 1 and are so arranged that when compared row with row there is always an equal number of changes and permanences of sign amongst the corresponding elements. Hence n is necessarily even. If we compare any two rows with a third row, the division of changes and permanences is again even. Hence n must be a multiple of 4. By a rearrangement of signs and order of columns we can always arrange any three rows in the form which for the case of $n = 12$ is

$$\begin{matrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & -1 & -1 \\ 1 & 1 & 1 & -1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 & -1 \end{matrix}$$

The actual maximum determinant is known for the following cases: (1) n a power of 2, (2) $n = 12$ or 20, (3) when the factors of n are any of the preceding numbers. For example, when n is 8, the determinant is

* *Bull. des Sciences math.*, 1893.