ON THE CANONICAL SUBSTITUTION IN THE HAMILTON-JACOBI CANONICAL SYSTEM OF DIFFERENTIAL EQUATIONS.

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LIE established a one-to-one correspondence between the integrals of the canonical system of differential equations and the one-parameter continuous groups of contact transformations of which the system admits, *i. e.*, making use of an integral of the system one can construct the infinitesimal transformation of a group of which the system admits, or, on the other hand by making use of the infinitesimal transformation of a group of which the system admits one can construct an integral of the system.* This theorem is the foundation of the modern transformation theory of dynamical systems.† The single canonical substitution (introduced by Jacobi) is of importance in the transformation and simplification of the dynamical equations.‡

The purpose of this paper is to define a type of the single canonical substitution which leads to an integral of the equations, *i. e.*, if one knows a member of the defined type of canonical substitutions one can construct an integral of the system using only algebraic operations.

A system of differential equations which has the form

$$(1)\quad \frac{d\pi}{dx}=\frac{\partial H}{\partial y},\quad \frac{dy}{dx}=-\frac{\partial H}{\partial \pi},\quad \frac{d\kappa}{dx}=\frac{\partial H}{\partial z},\quad \frac{dz}{dx}=-\frac{\partial H}{\partial \kappa}$$

is called a canonical system. H is a known function of y, π, z, κ and x; y, π, z and κ are the unknown functions; thus the solution of the system of four equations (1) consists in determining the four unknowns y, π, z and κ as such functions of x that the equations become identities in x. The functions y and π, as also z and κ, are called conjugate.

A substitution which leaves the form of the system (1) unchanged, though the function H may or may not be changed, is a canonical substitution.

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* Whittaker, Analytical Dynamics, page 308.

† Ibid., page 292.

‡ Poincaré, Mécanique céleste ; Jacobi, Vorlesungen über Dynamik.