

Let

$$n - m = k_1 r, \quad m - l = k_2 r$$

and

$$\therefore n = (k_1 + k_2)r + l, \quad m = k_2 r + l$$

$$x_2 = \epsilon x_1 \quad \text{where} \quad \epsilon^r = 1.$$

We may easily show that  $x_1$  is determined by the equation

$$(\epsilon^l - \epsilon)x_1 + \epsilon^l - 1 = 0,$$

and that  $x_1$  and  $x_2$  must be conjugate; and the investigation may be completed as in the last case.

It is obvious that the above method may be continued so as to include equations containing any number of terms.

It may be stated in conclusion that the problem solved in the present paper is connected with the more difficult problem of determining a quantity  $\rho$ , a function of  $a_0$  and  $a_1$ , such that there shall always be a root either of the equation  $f(x) = a$  or of the equation  $f(x) = b$ , with modulus less than  $\rho$ , and that this latter problem is connected with the theorem of Picard, which is discussed in Dr. Landau's paper.

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## ON THE DISTANCE FROM A POINT TO A SURFACE.

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It is well known that in order that the distance from a given point to a given surface be a maximum or a minimum it is necessary that this distance be measured on a normal to the surface. But, so far as I know, the various possible cases have not been enumerated. This is done in the following theorem :

If  $P$  be an elliptic point of a surface, and if  $C_1$  be the nearer and  $C_2$  the more remote of the principal centers of curvature, the distance from a given point  $N$  of the normal to  $P$  will be a minimum if  $N$  and  $P$  lie on the same side of  $C_1$ , a maximum if  $N$  and  $P$  lie on opposite sides of  $C_2$ , and neither a minimum nor a maximum if  $N$  coincide with  $C_1$  or  $C_2$ , or lie between them.