SHORTER NOTICES.

Handbuch der Theorie der Cylinderfunktionen. Von NIELS NIELSEN, Privatdozent an der Universität Kopenhagen, Inspektor des mathematischen Unterrichts an den Gymnasien Dänemarks. Leipzig, Teubner, 1904. xii + 408 pp.

DR. NIELSEN'S treatise contains twenty-seven chapters, of which all but four are largely devoted to an exposition of his own researches. While many of the results are not new, he has given more than a score of new equations and over half as many generalizations of known theorems, also numerous new proofs. Some of these proofs are for theorems stated but not proved by Lommel, Hurwitz, Jacobi, H. F. Weber and others. In several cases his proofs are intended to supersede less rigorous proofs by Sonin, Mehler and Ermakoff. Thus the proof by Bourget, that $J^n(x) = 0$ and $J^{n+p}(x) = 0$, if n and p are integers, have no common root, is declared to be valid only for the case of multiple roots, a proof also criticised by Rayleigh.

A cylindrical harmonic is defined as a solution of two functional equations which are shown to lead to Bessel's equation. The general solution of the first fundamental equation is obtained, and from a new property of the second equation follows a solution in the form of a continued fraction. The influence which Kepler's equation has exerted upon the study of cylindrical harmonics is recognized, but the problem from which the functions obtained their name appears to have been overlooked. The author would advise dropping the name Bessel function, because Bessel used only integral parameters, also because Bernoulli, Euler, Laplace, Fourier, Poisson and others had previously known of them; but he has consented to call J^n the Bessel cylindrical harmonic and Y^n the Neumann cylindrical harmonic because of the fundamental work done by their respective investigators.

Dr. Nielsen has aimed to obtain generalized forms and theorems, and with this in view he has devoted considerable space in the first of the four parts of his work to Lommel's Π function and to the similar Φ function, thus laying a firm foundation for a new theory of definite integrals with cylindrical harmonics and for Schlömilch's series. In this part are