$\lambda/(\lambda-1)$. Hence

$$(1)\quad q \neq 0,\ 1,\ \lambda,\ 1-\lambda,\ \frac{\lambda}{\lambda-1},\ \frac{1}{\lambda},\ \frac{1}{1-\lambda},\ \frac{\lambda-1}{\lambda}\quad (\lambda \neq 0,\ 1).$$

Here the six functions of λ are the six elements of the cross-ratio group, and each differs from 0 and 1. Hence equalities arise only when $\lambda = -1,\ 2$, or $\frac{1}{2}$, or when $\lambda^2-\lambda+1=0$.

When x_1, x_2, x_3 are distinct, $qx_1 + x_2$ is six valued only in the following cases: (i) one of the x's is an arithmetical mean between the other two, with $q \neq 0, 1, -1, 2, \frac{1}{2}$; (ii) $\Sigma x_j^2 = \Sigma x_j x_k$, with $q \neq 0, 1, \lambda, 1/\lambda$ (where $\lambda^2-\lambda+1=0$); (iii) neither of the relations on the x's holding, with q not equal to one of the eight distinct values (1).

It may now be readily shown that there exist six valued linear functions of the roots x_i of a cubic in the $GF[p^n]$ when $p^n > 8$; when $p^n = 7$; and when $p^n = 5$ or 8, with the x_i not all in the $GF[p^n]$.

5. In conclusion it may be remarked that the Galois theory as presented in Weber's Algebra may readily be extended to apply to modular fields, provided his argument on page 500 (of volume 1 of the second edition) be replaced by that in § 2 above.

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NOTE ON THE VARIATION OF THE DEFINITE INTEGRAL.

BY MR. N. J. LENNES.

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A function is said to be of limited variation on an interval ab if the set of sums

$$\left[\sum_{i=0}^{n-1} |f(x_i) - f(x_{i+1})|\right]$$

is bounded for the set of all partitions of ab. The points $a = x_0, x_1, x_2 \cdots, x_{n-1}, x_n = b$ of each partition are ordered on the interval according to the subscripts. The least upper