

analysis by which such product is represented. It is even independent of the kind of product. Thus if  $\mathbf{a}$  and  $\mathbf{b}$  are vectors representing length, the scalar product  $\mathbf{a} \cdot \mathbf{b}$ , the inner product  $[\mathbf{a} | \mathbf{b}]$ , the vector  $\mathbf{a} \times \mathbf{b}$ , the bivector  $[\mathbf{ab}]$ , the dyad  $\mathbf{ab}$ , the quaternion  $\mathbf{ab}$ , all have the dimensions  $[L]^2$ . It will not do for the physicist to lose sight of the fact for a moment. The followers of Grassmann, however, use the term dimension in a more geometric sense. If  $\mathbf{a}$ ,  $\mathbf{b}$ ,  $\mathbf{c}$  be vectors representing lengths,  $\mathbf{ab}$  is a bivector or vector of the second class (zweiter Stufe) and  $\mathbf{abc}$  is a trivector, a scalar, a quantity of the third class. Again, by the law of regressive multiplication,  $(\mathbf{abc})\mathbf{a}$  becomes a vector of the first class. As a matter of fact the physical dimensions of  $(\mathbf{abc})\mathbf{a}$  are  $[L]^4$ . As a geometer one may take great delight in the theory of geometric dimensions in multiple algebra, but as a physicist he must discard it as confusing. This is precisely what is done by the physicists who use Grassmann's notation.

The whole matter may, however, be let pass with good grace inasmuch as the book is otherwise singularly free from blemishes and well fulfils the requirements for an elementary text on geometric or multiple algebras. Jahnke's lectures supplemented first with the elementary geometric theory of imaginary numbers and of quaternions and second with Gibbs's theory of dyadics, where the ideas of products are considerably more extended than anywhere else, would form an excellent introductory course on higher algebra and would furnish, in addition, a very respectable knowledge of some parts of projective geometry, mechanics, and physics.

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### CELESTIAL MECHANICS.

*An Introduction to Celestial Mechanics.* By F. R. MOULTON, Ph.D., Instructor in Astronomy in the University of Chicago. New York, The Macmillan Company, 1902. xv + 384 pp.

THE need of a work written in the English language which would properly introduce astronomical students into the subject of celestial mechanics has been keenly felt for some time. In many of our universities the teaching of astronomy is confined to practical and to what is commonly called theoretical