NOTE ON THE STRUCTURE OF HYPER-COMPLEX NUMBER SYSTEMS.

BY DR. SAUL EPSTEEN.

(Read before the American Mathematical Society, September 7, 1905.)

IN the *Transactions* for April, 1905, on page 176 * the following theorem (No. III.) was enunciated:

Let $E, E_1, E_2, \cdots$ be a normal series of subalgebras of E (i. e., E_r is a maximal invariant subalgebra of E_{r-1}, $E_0 = E$) and let $K_1, K_2, \cdots$ be a series of complementary algebras, such that K_r accompanies E_{r-1} and is complementary to E_r. Under these assumptions the series $K_1 K_2, \cdots$ is, apart from the order, independent of the choice of the series $E, E_1, E_2, \cdots$. In other words, if $E, E_1', E_2', \cdots$ is any other normal series, the complementary series of algebras $K_1', K_2', \cdots$ which it defines is the same as the series $K_1, K_2, \cdots$, apart from the sequence.

With the exception of this theorem and the corresponding one where a *chief series* is used in place of the *normal series* all the proofs were made in a symbolic notation which is a generalization of one introduced by Frobenius in the theory of abstract groups. † In demonstrating the above proposition we made use of the *characteristic constants* $\gamma_{i_1i_2i_3}$ of the system

$$E \equiv e_1 \cdots e_n \qquad (e_{i_1}e_{i_2} = \sum_{i_3} \gamma_{i_1i_2i_3}e_{i_3}),$$

which, owing to the associative law, satisfy the n^3 conditions

$$\sum_{i_3}(\gamma_{i_1i_2i_3}\gamma_{i_3i_4i_5} - \gamma_{i_1i_3i_5}\gamma_{i_2i_4i_3}) = 0 \quad (i = 1, \cdots, n).$$

In the present note it is shown how this theorem can also be demonstrated in the symbolic notation without recourse to the γ's.

In order to avoid repetitions I employ the same numbering of theorems and equations, and also the same letters and symbols as in the above quoted paper, which this note supplements.

According to the demonstration of the theorem 1, we have (if $E_1 \neq E_1'$)

$$E = E_1 + E_1'.$$

* Epsteen-Wedderburn, "On the structure of hypercomplex number systems," *Transactions Amer. Math. Society*, vol. 6, pp. 172–178.

† Frobenius, *Berliner Sitzungsberichte*, 1895, p. 164.