define three mutually exclusive sets of subgroups, and these three together constitute the totality of the cyclic subgroups of $G_{\frac{1}{2}p(p^2-1)}$. *Their number is seen to be* $p^2 + p + 1$. *The order of the subgroup is* p *when* $\lambda \equiv 0$, *when* $4\mu\nu + 1 \equiv 0$, *and in the second pair; the order is* $\frac{1}{2}(p-1)$ *when* λ *or* $4\mu\nu + 1$ *is a quadratic residue of* p; *the order is* $\frac{1}{2}(p+1)$ *when* λ *or* $4\mu\nu + 1$ *is a non-residue of* p.

This explicit form of definition seems to be new, and on account of its simplicity may be of interest.

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EXTENSION OF A THEOREM DUE TO SYLOW.

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EVERY group G of order p^m, p being any prime number, contains at least p invariant operators. This fundamental theorem, due to Sylow,* is included in the following: *Every non-abelian group of order* p^m *contains at least* p *invariant commutator operators, and its commutator quotient group* † *is always non-cyclic.* In this connection it seems desirable to prove the following closely related theorems: It is possible to construct a non-abelian group having any arbitrary abelian group as a commutator quotient group. Every non-cyclic abelian group of order p^a is the commutator quotient group of some non-abelian group of order p^m.

The first part of the theorem in italics may be proved as follows: Let H represent the subgroup of G which is composed of its p^β invariant operators and let H_1 represent an invariant subgroup of order $p^{\beta+1}$ which includes H.‡ Any operator s of G which is not commutative with all the operators of H_1 transforms H_1 into a simple isomorphism with itself such that each of the operators of H corresponds to itself. Since H_1 is

* Sylow, *Math. Annalen*, vol. 5 (1872), p. 584.

† It seems convenient to speak of the quotient group corresponding to the commutator subgroup as the commutator quotient group.

‡ Every invariant subgroup of a group of order p^m is included in a larger invariant subgroup of arbitrary order less than p^m.