ON SELF-DUAL SCROLLS.

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THEOREM. *The necessary and sufficient condition that a scroll be self-dual is that it belong to a non-special linear complex.*

That the above condition is sufficient was stated, without proof, by Lie in the *Mathematische Annalen*, volume 5, page 179. Last year Professor Wilczynski proved the theorem by the use of differential equations. His proof is published in the *Mathematische Annalen*, volume 58, page 249. The following proof is algebraic.

Let the scroll belong to a non-special linear complex. The latter may be expressed in the form

$$p_{12} = \kappa p_{34} \qquad (\kappa \neq 0).$$

The equations of the scroll may be written

$$x_i = a_i(\lambda) + \mu a_i'(\lambda) \qquad (a_1' = a_4 = 0),$$

λ, μ being the Gaussian coördinates. But $p_{12} = a_1 a_2'$, $p_{34} = a_3 a_4'$, hence $a_1 a_2' = \kappa a_3 a_4'$ and we may replace the preceding equations by

$$x_1 = a_1(\lambda), \qquad x_2 = a_2(\lambda) + \mu a_3(\lambda),$$
$$x_3 = a_3(\lambda) + \mu a_3''(\lambda), \qquad x_4 = \mu\kappa a_1(\lambda).$$

This scroll may be defined as the locus of the intersection of corresponding planes of the cones

$$(1) \quad \kappa a_3 x_1 - a_1 \kappa x_3 + a_3'' x_4 = 0, \quad \kappa a_2 x_1 - a_1 \kappa x_2 + a_3 x_4 = 0.$$

The dual scroll is developed by the cones

$$(2) \quad a_1 x_1 + a_2 x_2 + a_3 x_3 = 0, \quad a_3 x_2 + a_3'' x_3 + \kappa a_1 x_4 = 0.$$

If we perform on (2) the operation

$$x_1 = -\kappa x_2, \quad x_2 = \kappa x_1, \quad x_3 = x_4, \quad x_4 = -x_3,$$