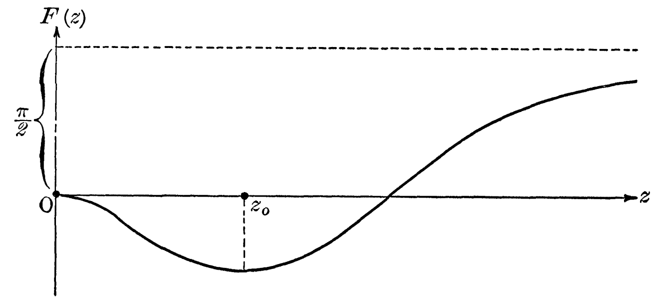
is negative for sufficiently small positive values of z, it follows that $F(z)$ decreases from the value zero to a certain negative minimum value, as z increases from 0 to z_0; then as z increases



from z_0 to $+\infty$, $F(z)$ increases continually and approaches for $z = +\infty$ the limit $+\pi/2$. Hence the equation (12) has one and but one real positive root.

This root being found, the equations (11), (10), and (9) yield a unique solution ω_0, ω_1, of (7) and (8), satisfying the inequality (4). Finally the values of h are z follow unambiguously from (3).

The existence and uniqueness of the solution of the proposed problem are therefore proved.

UNIVERSITY OF CHICAGO,
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ON THREE TYPES OF SURFACES OF THE THIRD ORDER REGARDED AS DOUBLE SURFACES OF TRANSLATION.

BY DR. A. S. GALE.

(Read before the American Mathematical Society, October 31, 1903.)

THIS note serves the double purpose of making a slight addition to the theory of three types of surfaces of the third order and of exhibiting the double surfaces of translation of lowest order. The latter surfaces enjoy all the properties of the double minimum surfaces * except those immediately dependent on the

* Lie, *Math. Annalen*, vol. 14 (1879), p. 346 et seq.; Darboux, Théorie des surfaces, vol. 1, p. 348 et seq.