The necessary condition that (1) and (2) have two integrals in common is that, in the following matrix obtained by successive differentiation,

$$\left\{\begin{matrix} \alpha_0 & \alpha_0' + \alpha_1 & \alpha_1' + \alpha_2 & \alpha_2' + \alpha_3 & \alpha_0' + \alpha_4 & \alpha_4' \\ 0 & \alpha_0 & \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \\ \beta_0 & 2\beta_0'+\beta_1 & \beta_0''+2\beta_1'+\beta_2 & \beta_1''+2\beta_2'+\beta_3 & \beta_2''+2\beta_3' & \beta_3'' \\ 0 & \beta_0 & \beta_0'+\beta_1 & \beta_1'+\beta_2 & \beta_2'+\beta_3 & \beta_3' \\ 0 & 0 & \beta_0 & \beta_1 & \beta_2 & \beta_3 \end{matrix}\right\},$$

the determinant consisting of the first five columns, and also that consisting of the first four columns and the sixth, shall vanish identically.

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TWO SYSTEMS OF SUBGROUPS OF THE QUATERNARY ABELIAN GROUP IN A GENERAL GALOIS FIELD.

BY PROFESSOR L. E. DICKSON.

(Read before the American Mathematical Society, August 31, 1903.)

1. Consider first the group G_ω composed of the

$$\omega = p^{4n}(p^{2n} - 1)(p^n - 1)$$

operators of the homogeneous quaternary abelian group in the $GF[p^n]$, $p > 2$, which multiply the variable η_1 by a constant. Those of its operators which leave ξ_1 and η_1 unaltered are given the notation

$$\begin{bmatrix} a & \gamma \\ \beta & \delta \end{bmatrix} : \quad \begin{matrix} \xi_2' = a\xi_2 + \gamma\eta_2, \\ \eta_2' = \beta\xi_2 + \delta\eta_2, \end{matrix} \qquad (a\delta - \beta\gamma = 1).$$

Certain other operators of G_ω are given the notation

$$[k, a, c, \gamma] = \left\{\begin{matrix} 1 & k & a & c \\ 0 & 1 & 0 & 0 \\ 0 & c - \gamma a & 1 & \gamma \\ 0 & -a & 0 & 1 \end{matrix}\right\}$$