the corresponding deformation of S leaves the lines of curvature unaltered and only in this case.

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ON INTEGRABILITY BY QUADRATURES.

BY DR. SAUL EPSTEEN.

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THE object of this note is to show that Vessiot's noted theorem that: "the necessary and sufficient condition that a linear differential equation shall be integrable by quadratures is that its group of rationality shall be integrable," * is a special case of the Jordan-Beke † theorem on reducibility of differential equations.

The Jordan-Beke theorem is to the effect that "if a linear differential equation is reducible in the sense of Frobenius ‡ then its group of rationality will transform a certain linear manifoldness of the solutions (which does not include the total n-dimensional manifoldness) into itself."

Analytically interpreted § this says that the group

$$(1)\quad \begin{aligned} y_1 &= a_{11}y_1 + \cdots + a_{1k}y_k, \\ &\cdots\cdots \\ y_k &= a_{k1}y_1 + \cdots + a_{kk}y_k, \\ y_{k+1} &= a_{k+1,1}y_1 + \cdots + a_{k+1,k}y_k + a_{k+1,\,k+1}y_{k+1} + \cdots + a_{k+1,n}y_n, \\ &\cdots\cdots \\ y_n &= a_{n1}y_1 + \cdots + a_{nk}y_k + a_{n,\,k+1}y_{k+1} + \cdots + a_{nn}y_n, \end{aligned}$$

is isomorphic with the group of rationality. For convenience it is well to adopt Loewy's notation, writing for (1) simply the coefficients

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* Vessiot: *Ann. de l'Ec. nor. sup.*, 1892.

† C. Jordan. *Bull. de la Soc. Math. de France*, vol. 2; Beke: *Math. Annalen*, vol. 45, p. 279.

‡ Frobenius: *Crelle*, vol. 76.

§ A. Loewy: "Ueber die irreduciblen Factoren," etc., *Berichte der math.-phy. Classe der Königl. Sächs. Gesellschaft der Wissenschaften zu Leipzig*, vol. 54 (1902), pp. 1–13.