itself is commutative with every operator of G_i. Let H_1 be the commutator subgroup. The group $\{H_1, G_i\}$ is of order $p_1^{\beta} p_i^{\alpha_i}$. This contains $p_1^{\gamma} (\gamma \leqq \beta)$ subgroups of order $p_i^{\alpha_i}$, and therefore $p_1^{\gamma} \equiv 1 \pmod{p_i}$. Hence if

$$p_1^{\gamma} \not\equiv 1 \pmod{p_i} \quad (0 < \gamma \leqq \beta),$$

every commutator is commutative with every operator of G_i. Then $A_j^{-1} A_i A_j = A_i t_i$, where A_j is any operator of

$$G_j \qquad (j = 1, 2, \cdots, n)$$

and A_i is any operator of G_i; and $A_j^{-1} A_i^{p_1^{\beta_i}} A_j = A_i^{p_1^{\beta_i}}$, where $p_1^{\beta_i}$ is the order of t_i. But $p_1^{\beta_i}$ is relatively prime to p_i. Therefore $A_j^{-1} A_i A_j = A_i$, and G is the direct product of the groups G_j."

THEOREM. *If a group G of order $p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_n^{\alpha_n}$ ($p_1, p_2, \cdots, p_n$ being distinct primes) has a commutator subgroup of order p_1^{β} and if $p_1^{\gamma} \not\equiv 1 \pmod{p_i}$ $(0 < \gamma \leqq \beta)$, $(i = 2, 3, \cdots n)$, then G is the direct product of groups of orders $p_1^{\alpha_1}, p_2^{\alpha_2}, \cdots, p_n^{\alpha_n}$ respectively.*

CORNELL UNIVERSITY.
August, 1902.

NOTE ON IRREGULAR DETERMINANTS.

BY PROFESSOR L. I. HEWES.

IN Gauss's table * of binary quadratic forms the two negative determinants -468 and -931 of the first thousand are classed as regular and their genera and classes given correctly. Perott † has pointed out that these two determinants are irregular. The details of the classes of the original thirteen irregular determinants of Gauss have been worked out by Cayley ‡ and on the following page are given the details, in his notation, for the properly primitive reduced forms of the two determinants added by Perott's investigation.

* C. F. Gauss, Werke, vol II, p 450.
† "Sur la formation des déterminants irreguliers," *Crelle*, vol. 59.
‡ Cayley's Collected Papers, vol. 5, p. 141.