ON THE FORMS OF QUINTIC SCROLLS.

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THE only paper devoted to the systematic study of quintic scrolls is that by Schwarz,* in which surfaces are classified according to the configuration of the double curve. Fifteen types are enumerated, of which ten are unicursal, four of genus 1, and one of genus 2. Schwarz regarded the scroll as generated by the line of intersection of corresponding planes in two developables the sum of whose classes is 5.

It seems desirable to enumerate the types from the dual standpoint, *i. e.*, regarded as generated by the line joining corresponding points of two plane or twisted curves. Of course, the same forms will appear that were obtained before, but the relations between subforms are more clearly brought out.

A. Unicursal Quintics.

Let a line δ and a unicursal quartic c_4 be put in (1, 1) correspondence and lines joining corresponding points be drawn; these lines will generate a quintic scroll having δ for a simple director line; any plane through δ will cut the scroll in 4 generators which intersect in 6 points and each cuts δ in one point. The plane is a fourfold tangent plane; hence the double curve is of order 6 (type II).† The double curve does not cut δ.

Let δ cut c_4, then a generator coincides with δ; the residual curve is of order 5, it cuts δ in 3 points (VI).

Let δ cut c_4 twice, then δ is a triple line; the residual curve is of order 3, it cuts δ in 2 points (III).

Let δ cut c_4 three times; δ counts as a fourfold line and there is no other nodal curve on the surface (I).

Let c_4 be replaced by a double conic c_2^2; the double curve consists of the conic and a twisted quartic which is cut by every generator twice (VIII).

Thus far the correspondence is between a line and a curve of order 4; it remains to consider the correspondence between a conic c_2 and a unicursal cubic c_3. The general case gives nothing new from the standpoint of the double curve.

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* "Ueber die geradlinigen Flächen fünften Grades" *Crelle*, vol. 67.

† The Roman numerals refer to types given by Schwarz.