

$$\Delta(\varphi\psi) = \varphi\Delta\psi + \psi\Delta\varphi + 2\Omega,$$

where

$$\Omega = \frac{\partial\varphi}{\partial x} \frac{\partial\psi}{\partial x} + \frac{\partial\varphi}{\partial y} \frac{\partial\psi}{\partial y} + \frac{\partial\varphi}{\partial z} \frac{\partial\psi}{\partial z}.$$

As is well known, Ω vanishes only when the two families $\varphi = \text{const.}$ $\psi = \text{const.}$, are orthogonal. Therefore

IX. *If a pair of potential surfaces $\varphi = 0$, $\psi = 0$ combined form a potential surface, the families $\varphi = \text{const.}$, $\psi = \text{const.}$ are orthogonal.*

The projective generalization of this result is

IX'. *If a surface apolar to a conic decomposes into two surfaces apolar to the same conic, then the tangent planes to the latter surfaces at any point in their intersection cut the plane of the conic in a pair of lines conjugate with respect to the conic.*

A MODERN ENGLISH CALCULUS.

An Elementary Treatise on the Calculus, with Illustrations from Geometry, Mechanics and Physics. By GEORGE A. GIBSON, M.A., F.R.S.E., Professor of Mathematics in the Glasgow and West of Scotland Technical College. London, Macmillan & Co., 1901. 12mo, pp. xix + 459.

In the year 1891 Harnack's Elements of the Differential and Integral Calculus, which appeared in Leipzig in 1881, was translated into English. This book gave the first systematic presentation in the English language of the leading principles of modern analysis in their relation to the foundations of the infinitesimal calculus. While not wholly free from errors, and sometimes difficult to read, owing to inadequate exposition of details, the book is nevertheless conceived in the spirit of modern mathematics and it lays stress on those principles of analysis which are essential for a rigorous development of the calculus.

The first book of English origin to treat the calculus from a modern standpoint was Lamb's Infinitesimal Calculus,* published in 1897. This is an excellent treatise and any later work on the calculus, of modern tendencies, must have many points of contact with it.

* A notice of this book by the present writer appeared in *Science*, new series, vol. 7 (1898), No. 176, p. 678.