THE APPLICATION OF THE FUNDAMENTAL LAWS OF ALGEBRA TO THE MULTIPLICATION OF INFINITE SERIES.

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THE present writer has given examples in which an absolutely convergent series is obtained as the result of multiplying two conditionally convergent series together, or of multiplying one conditionally convergent series by a divergent series.*

He has also given an example of two divergent series whose product is absolutely convergent.†

Pringsheim has treated this subject from a more general point of view and by very simple methods has shown that the property in question is typical of certain classes of series.‡

In the present paper it is proposed to establish a class of series with real terms, possessing the property alluded to, but which seems to be distinct from the class given by Pringsheim. Next, we shall consider the validity of the fundamental laws of algebra in the multiplication of infinite series. Then, with aid of our conclusions relating to these laws, we shall point out another method for obtaining divergent series whose product is absolutely convergent. Lastly we shall generalize a theorem of Abel on the multiplication of series.

§1.

In the series S_1 and S_2, obtained respectively by removing the parentheses from the series

$$S_1' \equiv \sum_{v=0}^{v=\infty} (a_{4v} - a_{4v+1} + a_{4v+2} - a_{4v+3}),$$

$$S_2' \equiv \sum_{v=0}^{v=\infty} (b_{4v} + b_{4v+1} - b_{4v+2} - b_{4v+3}),$$

wherein the a's and b's are real and positive, let the following conditions be satisfied :

(1) The vth term in S_1 and in S_2 shall be $\gtreqless v^{-r}$, where $\frac{1}{2} < r \gtreqless 1$, but $\sum a_s$ and $\sum b_s$ are both divergent.

* *Trans. Amer. Math. Society*, vol. 2, pp. 25–36, January, 1901.

† *Science*, new ser., vol. 14, p. 395 (September 13, 1901)

‡ *Trans. Amer. Math. Society*, vol. 2, pp. 404–412, October, 1901.