ON THE ABELIAN GROUPS WHICH ARE CONFORMAL WITH NON-ABELIAN GROUPS.

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Two distinct groups are said to be conformal when they contain the same number of operators of each order. * The present paper is devoted to the determination of all the abelian groups which are conformal with non-abelian groups. The complete solution of the converse of this problem, *viz.*, the determination of all the non-abelian groups which are conformal with abelian ones is much more difficult, since a large number of distinct non-abelian groups may be conformal with the same abelian group while no more than one abelian group can be conformal with one non-abelian group. In fact, two distinct abelian groups cannot be conformal.

It is well known that there is only one group of order 2^m which does not include any operator of order 4, *viz.*, the group of type $(1, 1, 1, \cdots\cdots)$. † Moreover, there is only one cyclic group of order 2^m, and when $m < 4$ no two groups of order 2^m are conformal. We proceed to prove that every abelian group G of order 2^m which does not satisfy one of these conditions is conformal with at least one non-abelian group.

Let H be the subgroup of G which is generated by the square of one of its independent generators s of lowest order together with all the other independent generators of G. The order of H is 2^{m-1}. Since $m > 3$ there is an operator t of order 2 which has the following properties. ‡ It transforms H into itself, it is commutative with half of the operators of H (including all those which are not of highest order), and it transforms the rest into themselves multiplied by an operator of order 2 which is not the square of a non-invariant operator of H; *i. e.*, t does not transform an operator of order 4 contained in H into its inverse. The non-abelian group generated by H and t is conformal with G whenever $s^2 = 1$.

When the order of s exceeds two, we may make the group generated by t and H (written as a regular substitution group) simply isomorphic with itself by writing it in two

* *Quar. Jour. of Math.*, vol. 28 (1896), p. 270.

† *Ibid.*, p. 208.

‡ BULLETIN, vol. 5 (1898), p. 245; also vol. 6 (1899), p. 236.