NOTE ON GEOMETRY OF FOUR DIMENSIONS.

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(Read before the American Mathematical Society, April 28, 1900.)

1. Speculations relative to the geometry of n dimensional space have followed several fairly well-defined trends which not infrequently cross one another: 1° A direct extension of the Cartesian geometry, which extension is to be regarded as nothing more than a convenient form of phraseology; in this form did n dimensional space spring forth from the minds of Grassmann, Cayley, Gauss, and Cauchy, and the idea was likely familiar to Euler and Lagrange. 2° The transformation of the ordinary visualizable spaces of two and three dimensions into manifoldnesses of higher or lower dimensions by introducing space elements other than the point or its dual element; for example, the line geometry of Plücker, the sphere geometry of Lie, the five dimensional manifoldness of all conics in the plane as an auxiliary to Ball's theory of screws; this category becomes more concrete perhaps than any other. 3° The absolute geometry of space; here would appear the celebrated dissertation of Riemann, the well-known memoirs of Helmholtz and Lie and the elaborate treatise of Veronese. 4° The extension of the methods of ordinary differential geometry to spaces of many dimensions; to this class belong the works of Christoffel, Beltrami, Bianchi, Cesàro, and Ricci, and the quite recent contributions of Darboux and his pupils. 5° The direct extension of the concepts and problems of metrical and projective geometry of ordinary space, as exemplified in the memoirs of Jordan, d'Ovidio, and Veronese. 6° The theory of birational correspondences between n dimensional aggregates as studied by Noether, Kantor, and Brill. 7° The descriptive geometry of space of n dimensions as begun in the papers of Veronese, Stringham, Schlegel, and Segre. 8° The kinematics of higher spaces as developed by Jordan, Clifford, and Beltrami. 9° The interpretation of n dimensional geometry in the light of the theory of groups as exhibited by Lie, Klein, and Poincaré.

2. It is proposed here to make an expository contribution to the ninth and fourth of the above categories, constructing ordinary four dimensional space by the method of Lie's theory of continuous groups, and studying curves of triple