SOME REMARKS ON TETRAHEDRAL GEOMETRY.

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(Read before the American Mathematical Society, June 27, 1900.)

1. Any two quadric surfaces have in general a common self-conjugate tetrahedron. If we refer to it a system of homogeneous point coördinates x_1, x_2, x_3, x_4, the equations of the two quadrics take the forms:

$$(1)\qquad \begin{aligned} g_1x_1^2 + g_2x_2^2 + g_3x_3^2 + g_4x_4^2 &= 0, \\ h_1x_1^2 + h_2x_2^2 + h_3x_3^2 + h_4x_4^2 &= 0. \end{aligned}$$

Now the polar planes of a point with respect to these two quadrics cut each other in a straight line, which we may also find as the polar line of the given point in the following way, starting from the quartic curve of intersection of the two quadrics: We draw the two chords of the quartic curve which pass through the point, and determine on each of them the fourth harmonic to its two curve points and the given point. Joining these two fourth harmonic points we have at once the required polar line.

We fix a straight line in space by Plückerian line coördinates, viz., if x_i, y_i be the coördinates of any two points upon the line, by the six expressions

$$(2)\qquad p_{ik} = x_iy_k - x_ky_i.$$

Between them the identical relation exists

$$(3)\qquad p_{23}p_{14} + p_{31}p_{24} + p_{12}p_{34} = 0.$$

The polar lines of all points in space, with respect to a quartic space curve of the first species, form a complex of lines. This complex has been called by Reye a tetrahedral complex. If z_i be the coördinates of the point, we immediately find for the coördinates of its polar line

$$(4)\qquad p_{ik} = g_{ik}z_iz_k,$$

by making

$$(5)\qquad g_{ik} = g_ih_k - g_kh_i.$$

These magnitudes g_{ik}, which also satisfy the condition

$$(6)\qquad g_{23}g_{14} + g_{31}g_{24} + g_{12}g_{34} = 0$$