SOME THEOREMS CONCERNING LINEAR DIFFERENTIAL EQUATIONS OF THE SECOND ORDER.

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I WISH to communicate to the Society certain results at which I have arrived, reserving the proofs and also further developments for another occasion. We shall be concerned with differential equations whose coefficients are not assumed to be analytic, and we consider the case in which $x = 0$ is a *regular singular point** of the equation, that is, the case in which the equation is of the form:

$$(1) \qquad \frac{d^2y}{dx^2} + \left(\frac{\mu}{x} + p_1\right)\frac{dy}{dx} + \left(\frac{\nu}{x^2} + q_1\right)y = 0,$$

where μ and ν are constants, and $|p_1|$ and $x\,|q_1|$ can be integrated up to the point $x = 0$.† For the sake of simplicity I shall not here refer to the case in which the exponents of the point $x = 0$ are equal, although this case might easily be included. By methods analogous to those which I have sketched in a simpler case (BULLETIN, Second Series, Volume 6, p. 100) we obtain the

FIRST THEOREM OF COMPARISON: *If the exponents of $x = 0$ in (1) are real and unequal, and y_1 is the solution of (1) which corresponds to the larger exponent, and if y_1 vanishes when $x = x_1 > 0$, but does not vanish between 0 and x_1; if moreover we have a second equation of the same form as (1) which differs from it only in that throughout the interval $0 < x \leqq x_1$ the coefficient of y in the second equation $\geqq \nu/x^2 + q_1$ (the equality sign not holding throughout the whole interval); and finally if y_2 is the solution of this second equation which corresponds to the larger exponent of $x = 0$; then y_2 has at least one root in the interval $0 < x < x_1$.*

In order to compare the solutions corresponding to the smaller exponents, we must restrict our differential equation further by assuming that a constant $k < 1$ can be found such that $x^k p_1$ and $x^{k+1} q_1$ remain finite as x approaches zero. We will refer to this as *restriction* (A). We then have the

THEOREM: *The solution of (1) corresponding to the exponent*

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* For the meaning of this and other terms used, see my paper in the January number of the *Transactions*, p. 40; or an earlier paper in the BULLETIN, 2d Series, vol. 5, p. 275.

† For a more precise statement see my papers just referred to.