NOTE ON NAPIER'S RULES OF CIRCULAR PARTS.

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THE object of this note is to call attention to Napier's conception of Napier's rules of circular parts in the theory of the right spherical triangle. Text-books and treatises are considerably at variance as to the importance and status of these rules. They are continually referred to as mere mnemonics requiring independent proof in each case; and not infrequently their rôle in this capacity is discouraged, analogies with corresponding formulæ in the plane being offered as preferable substitutes. What follows shows that these much abused individuals are entitled to more generous consideration and should be invested with the dignity of a theorem. The circumstance of their various misinterpretations emphasizes again that the student of mathematics should trust no middle man, but go with his own head to original sources, to the masters themselves. Second-hand ideas are as full of bacteria as are second-hand books and clothes.

Let ABC_1 be a right spherical triangle whose right angle is at C_1; draw the great circles which coincide with the sides of the triangle and those whose poles are the vertices A and B. These five great circles form a spherical pentagon surmounted by five right spherical triangles. The uniformity of Napier's rules lies in the fact that all five of these associated triangles, though possessing different natural parts, have one and the same set of circular parts.

A little attention to the geometry of this beautiful figure yields the following geometrical interpretations of the rules of Napier, the first two of which were given by Napier in the original presentation of the rules,* and the first of which was rediscovered by Ellis.†

Napier's rules express 1° properties common to the members of the system of five right-angles T_1, T_2, T_3, T_4, T_5; 2° characteristics which the individuals of the family of five quadrantal triangles ABD, $\cdots$, have in common; 3° relations among the parts of the self-polar pentagon $ABDEF$; 4° relations in the elements of the pentagon $C_1C_2C_3C_4C_5$; 5° relations among the sides and angles of the pentagon

*See Napier—Logarithmorum canonis descriptio, Lugduni, apud Barth. Vincentium, MDCXX, Lib. II., Cap. IV., pp. 31 et seq.

†See The mathematical and other writings of Robert Leslie Ellis, edited by William Walton, Cambridge, 1863, pp. 328 et seq.