close dependence of the operations of subtraction and multiplication of numbers on the same operations for lines, it would seem that there must be some connection.

It is evident that the Greeks were even farther from a continuum of numbers than they were from a continuum of their line symbols. For no number in their system existed which expressed their incommensurable lines, not to mention the countless kinds of incommensurability of which they knew nothing.

These considerations indicate that Greek mathematics rested on a very narrow basis so long as it clung to its line notation. The sense of rigor, as shown by postulating the existence of a product of two factors certainly would not allow them to assume a continuous system, as less careful mathematicians have done. This line notation did not admit of sufficient expansion to allow them to establish on *that* such a system. Thus, until the foundation of their mathematical science was utterly changed, an advance to algebra and calculus was impossible.

YALE UNIVERSITY,
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MAXIMA AND MINIMA OF FUNCTIONS OF SEVERAL VARIABLES.

BY PROFESSOR JAMES PIERPONT.

IN treating the theory of maxima and minima in my lectures this year I have been astonished to find that the presentation of this theory in all English and American text-books on the calculus which I could consult was false. That the older editions of such standard treatises as Todhunter, Williamson, and Byerly should be wrong in this particular was not astonishing since it was only in 1884 that Peano in his critical notes to the Calcolo Differenziale of Genocchi called attention to the point in question. Since then L. Scheeffers,* O. Stolz,† and von Dantscher‡ have devoted memoirs to this interesting but difficult subject and their results have found a place in the new edition of the Cours d'Analyse of C. Jordan and the Grundzüge of O. Stolz.

* *Mathematische Annalen*, vol. 35, p. 541.
† *Sitzungsberichte Vienna Academy*, 1890 (June).
‡ *Annalen*, vol. 42, p. 89.