substitutions have the same composition formula as linear fractional substitutions. Hence, according as -1 is a square or a not-square, H' is simply isomorphic to the "real" or the "imaginary" form* of the group of linear fractional substitutions of determinant unity. Thus, for $p^n > 3$, H' is simple.

15. Observing that the squares of the substitutions

$$O_{1,2}^{\alpha,\beta}, \quad O_{1,2}^{\alpha,\beta}\, T_{13} C_1 C_2 C_3, \quad O_{1,2}^{\alpha,\beta}\, T_{13} T_{24}$$

are respectively $Q_{1,2}^{\alpha,-\beta}$, $O_{1,2}^{\alpha,\beta}\, O_{3,2}^{\alpha,\beta}$, $O_{1,2}^{\alpha,\beta}\, O_{3,4}^{\alpha,\beta}$,
we may unite our results into the following

THEOREM: *The squares of the linear substitutions on m indices in the $GF[p^n]$, $p \neq 2$, which leave invariant the sum of the squares of the m indices, generate a group, which for $m = 2k + 1$ has the order*

$$\tfrac{1}{2}(p^{2nk} - 1)\, p^{2nk-n}\, (p^{2nk-2n} - 1)\, p^{2nk-3n} \cdots (p^{2n} - 1)\, p^n$$

and is simple except when $p^n = 3$, $m = 3$; while for $m = 2k > 4$ it has the factors of composition 2 and

$$\tfrac{1}{4}[p^{nk} - (\pm 1)^k]\, p^{nk-n}\, (p^{2nk-2n} - 1)\, p^{2nk-3n} \cdots (p^{2n} - 1)\, p^n,$$

the sign $\pm$ depending upon the form $4l \pm 1$ of p^n.

UNIVERSITY OF CALIFORNIA,
February 10, 1898.

A PROOF OF THE THEOREM: $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

BY MR. J. K. WHITTEMORE.

(Read before the American Mathematical Society at the Meeting of April 30, 1898.)

THEOREM: *Let $u = f(x, y)$ denote a function of the two independent variables x and y which, together with its first derivatives and the two second derivatives in question, is continuous in the neighborhood of the point (x, y); then* $\dfrac{\partial^2 u}{\partial x \partial y} = \dfrac{\partial^2 u}{\partial y \partial x}$

Let $\dfrac{\partial^2 f(x, y)}{\partial x \partial y}$ denote $\dfrac{\partial}{\partial x}\left(\dfrac{\partial f(x, y)}{\partial y}\right)$

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* Moore: Mathematical Papers of the Chicago Congress (1893), "A doubly-infinite system of simple groups," §§ 5–6.