NOTE ON HYPERELLIPTIC INTEGRALS.

BY PROFESSOR ALEXANDER S. CHESSIN.

(Read before the American Mathematical Society at the Meeting of October 30, 1897.)

LET X_r denote a polynomial in x of degree r; $P_m(x)$, $Q_n(x)$, $\cdots$ polynomials in x of degrees m, n, $\cdots$. We know that the integration of

$$\int f(x, \sqrt{X_r})\, dx,$$

where $f(x, \sqrt{X_r})$ is a rational function of x and $\sqrt{X_r}$, is reduced to the integration of

$$(1) \qquad \int \frac{R(x)\,dx}{\sqrt{X_r}}$$

where $R(x)$ is a rational function of x. This note is intended to give a practical rule for the integration of (1).

Let

$$(2) \qquad R(x) = \frac{P_m(x)}{\prod\limits_{x=1}^{k=s}(x-a_k)^{n_k}}$$

We may assume that $P_m(x)$ has no factor $x - a_k$, otherwise the common factors may be cancelled. We also assume that all the factors of X_r are simple, for double factors could be taken outside the radical.

Suppose first that none of the a_k are roots of $X_r = 0$. Then we have the equality

$$(3) \qquad \int \frac{P_m(x)\,dx}{\prod\limits_{k=1}^{k=s}(x-a_k)^{n_k}\sqrt{X_r}} = \frac{Q_p(x)\sqrt{X_r}}{\prod\limits_{k=1}^{k=s}(x-a_k)^{n-1}}$$

$$+ \sum_{k=0}^{k=r-2}\lambda_k \int \frac{x^k dx}{\sqrt{X_r}} + \sum_{k=1}^{k=s}\mu_k \int \frac{dx}{(x-a_k)\sqrt{X_r}}$$

where