

on the structure of the substitution group. Similarly Lie makes the integration of a system of partial differential equations which admits of a finite continuous group of transformations depend upon the integration of a series of auxiliary systems, and the number of these systems, their nature and the way in which they are related to one another depends on the structure of the continuous group. The importance of the structure of finite continuous groups is further illustrated in Picard's theory of linear differential equations and more generally in the theory of differential equations which admit of a fundamental system of integrals.

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A GEOMETRICAL LOCUS CONNECTED WITH A SYSTEM OF COAXIAL CIRCLES.

BY PROFESSOR THOMAS F. HOLGATE.

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SUPPOSE there be given a sheaf or pencil of rays whose centre is P and a system of coaxial circles lying in the same plane. Two rays of the sheaf will be tangent to each circle of the system and two circles of the system will be touched by each ray of the sheaf. If we start with any one circle of the system k_1 and one of its tangent rays m_1 , it is easy to determine the second circle k_2 to which this ray is tangent, then the second ray m_2 tangent to k_2 , then the second circle k_3 to which m_2 is tangent and so on indefinitely, the circles and rays forming a continuous chain. Whether or not this chain of circles and rays will return into itself depends upon the location of the centre P with respect to the system of coaxial circles and I undertake in the present paper to find the locus of the point P for which the chain of circles and rays will close with three circles and three rays.

In other words, I undertake to find the locus of points through which three lines can be drawn tangent to three circles of a coaxial system in pairs.

Any two circles and a pair of common tangents form such a closed circuit containing two elements of each kind and the point P may be any one of the six vertices of the complete quadrilateral formed by the four tangents common to the two chosen circles.