Let $n = 5$; then $q = 1$. An arbitrary function of the determinant

$$\triangle \equiv \begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 12 & 13 & 14 & 15 \\ 1 & 21 & 0 & 23 & 24 & 25 \\ 1 & 31 & 32 & 0 & 34 & 35 \\ 1 & 41 & 42 & 43 & 0 & 45 \\ 1 & 51 & 52 & 53 & 54 & 0 \end{vmatrix},$$

where $12, 13, \cdots$, have the signification given by the identity and equation (2), is a general solution of the simultaneous system (3) for $n = 5$. In particular the vanishing of $\Delta$ satisfies the system (3) and hence expresses the relation among the mutual distances of five points in space, a result known to Lagrange. The fifth order determinant $\Delta_0$, the minor of $\Delta$ with regard to the upper left hand corner element, equated to zero expresses the necessary and sufficient condition that five points be on a sphere. Similarly the vanishing of $\Delta_{00}$ and that of $\Delta_{000}$ give the conditions, respectively, that four points be coplanar and three points collinear.

Construct the determinant $\Delta$ for $n$ points and call it $D$. $D = 0$ is then a generalization of the theorem of Lagrange expressed by $\Delta = 0$. This extension is warranted by the form of (3), the symmetry of $\Delta$, and the fact that the invariants considered are absolute invariants.

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# NOTE ON THE FUNDAMENTAL THEOREMS OF LIE'S THEORY OF CONTINUOUS GROUPS.

BY DR. EDGAR ODELL LOVETT.

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Lie's theory of continuous groups rests upon the following three fundamental theorems:*

* See LIE: *Vorlesungen über continuierliche Gruppen*, herausgegeben von Scheffers, Leipzig, 1893, chapter XV; LIE: *Theorie der Transformationsgruppen*, bearbeitet unter Mitwirkung von Engel, Leipzig, Erster Abschnitt, 1888, chapters II, IV, IX, XVII; zweiter Abschnitt, 1890, chapter XVII; dritter Abschnitt, 1893, chapter XXV.