LINES COMMON TO FOUR LINEAR COMPLEXES.

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In his discussion of the invariants of one or more linear complexes, Klein* makes the statement that four such complexes have two lines in common, which become coincident when the combinant of the four complexes vanishes, but otherwise the reality of the lines is not discussed.

The corresponding criterion for spherical geometry is of value in studying the cyclides and it can be proved by geometrical methods that the spheres common to four linear spherical complexes are real when the combinant is negative. On account of the direct interpretation of the simpler invariants from one geometry into the other, one might conclude by analogy that the same law holds here, which, however, is not the case.

For convenience, transform the quadratic relation

$$P_{12}P_{34} + P_{13}P_{42} + P_{14}P_{23} = 0$$

by the transformation

$$P_{12} = x_1 + x_2,\quad P_{13} = x_3 + x_4,\quad P_{14} = x_5 + x_6,$$
$$P_{34} = x_1 - x_2,\quad P_{42} = x_3 - x_4,\quad P_{23} = x_5 - x_6, \text{ into}$$
$$x_1^2 - x_2^2 + x_3^2 - x_4^2 + x_5^2 - x_6^2 = 0.$$

Let the four given complexes be

$$(1)\qquad \psi_i \equiv a_i x_1 + b_i x_2 + c_i x_3 + d_i x_4 + e_i x_5 + f_i x_6 = 0 \; [i = 1, 2, 3, 4].$$

The invariant of ψ_i is

$$(2)\qquad A_{ii} \equiv a_i^2 - b_i^2 + c_i^2 - d_i^2 + e_i^2 - f_i^2$$

and the simultaneous invariant of ψ_i, ψ_k is

$$(3)\qquad A_{ik} \equiv a_i a_k - b_i b_k + c_i c_k - d_i d_k + e_i e_k - f_i f_k.$$

Two complexes are in involution when their simultaneous invariant vanishes; a general complex is in involution with

* "Differentialgleichungen in Liniengeometrie," *Math. Annalen*, vol. 5.