THE ARITHMETIZING OF MATHEMATICS.*

An Address delivered at the public meeting of the Royal Academy of Sciences of Göttingen, November 2nd, 1895.

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THOUGH the details of mathematical science, by their very nature, elude the comprehension of the layman, and therefore fail to arouse his interest, yet the mathematician may profitably indicate certain general points of view from which he surveys his science, especially if these points of view determine his attitude to kindred subjects. I propose therefore on the present occasion to explain my position in regard to an important mathematical tendency which has as its chief exponent Weierstrass, whose eightieth birthday we have lately celebrated. I refer to the *arithmetizing* of mathematics. Some account of this tendency and its origin may be given by way of preface.

The popular conception of mathematics is that of a strictly logically coördinated system, complete in itself, such as we meet with, for instance, in Euclid's geometry; but as a matter of fact, modern mathematics in its origin was of a totally different character. With the contemplation of nature as starting point, and its interpretation as object, a philosophical principle, the principle of continuity, was made fundamental; and the use of this principle characterizes the work of the great pioneers, Newton and Leibnitz, and the mathematicians of the whole of the eighteenth century—a century of discoveries in the evolution of mathematics. Gradually, however, a more critical spirit asserted itself and demanded a logical justification for the innovations made with such assurance, the establishment, as it were, of law and order after the long and victorious campaign. This was the time of Gauss and Abel, of Cauchy and Dirichlet. But this was not the end of the matter. Gauss, taking for granted the continuity of space, unhesitatingly used space intuition as a basis for his proofs; but closer investigation showed not only that many special points still needed proof, but also that space intuition had led to the too hasty assumption of the generality of certain theorems which are by no means general. Hence arose the demand for exclusively arithmetical methods of proof; nothing shall be

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