tions between the parameters defining the system. Finally, the conditions determined by three relations form the third group and are all conditions of constraint without friction.

This general theory is followed by numerous applications worked out completely, and with the elegance and clearness which are characteristic of the two courses of lectures we have before us.

It is only fair in concluding this review to remark to Mr. Hermann's credit that the reading of these two volumes is not in the slightest degree trying to the eyes, which unfortunately could not be said with regard to, for instance, Mr. Klein's lithographed courses.

ALEXANDRE S. CHESSIN.

JOHNS HOPKINS UNIVERSITY,
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A GEOMETRIC PROOF OF A FUNDAMENTAL THEOREM CONCERNING UNICURSAL CURVES.

BY PROFESSOR W. F. OSGOOD.

1. IF $f(x,y)=0$ is the equation of an irreducible curve of deficiency 0, then, as is well known, the coördinates can be expressed as rational functions of a parameter λ:*

$$x=r_1(\lambda) \qquad y=r_2(\lambda)$$

where not only to a given value of λ corresponds one and only one point of the curve, but conversely to a given point (x,y) on the curve corresponds in general one and only one value of λ.† λ can be expressed as a rational function of x and y.

If a multiple-leaved Riemann surface spread out, say, over the x-plane be used to represent geometrically the above locus, $f(x,y)=0$, the deficiency of this surface will likewise be 0, and as is shown in the elements of Riemann's theory of functions,‡ there exist single-valued functions on such a surface having but a single pole, and that of the first order, and taking on every value once and only once on the surface. Call such a function λ. Being single-valued on the surface it will be a rational function of x and y: $\lambda=R(x,y)$,

*SALMON, *Higher Plane Curves*, p. 30; CLEBSCH-LINDEMANN, *Geometrie* vol. I., p. 883.

† CAYLEY has given to such curves the name *unicursal*.

‡ KLEIN, *Modulfunctionen*, vol. I., p. 493 et seq. PICARD, *Traité d' analyse*, vol. II., Ch. XVI.