

THE THREE GREAT PROBLEMS OF ANTIQUITY,
CONSIDERED IN THE LIGHT OF MODERN
MATHEMATICAL RESEARCH.

Vorträge über ausgewählte Fragen der Elementargeometrie. F. KLEIN. Ausgearbeitet von F. TÄGERT. Leipzig, Teubner, 1895. pp. 66.

AMONG the minor mathematical works published during the past year, one of the most interesting is Klein's Festschrift for the third meeting of the association for the advancement of mathematical and scientific teaching in the Gymnasia. The author has himself explained, in his paper *Ueber Arithmetisirung der Mathematik*, that his main object in writing this pamphlet was to emphasize the necessity for strict logical developments as a corrective to the tendency to rely too exclusively on intuitive proofs. Intended primarily for teachers of the more elementary mathematics, this pamphlet is confined to the three great problems of antiquity, as they appear in the light of modern research—or rather to the mathematical investigations for which these problems have furnished the text. These problems, (1) the duplication of the cube, (2) the trisection of an angle, (3) the quadrature of the circle, presented themselves at a very early stage in the development of mathematics, and they naturally present themselves correspondingly early in the mathematical development of the individual.

The Greek mathematicians, striving to solve these problems—problems for each of which the solution was impossible, within the domain of the geometry of the straight line and circle—were led to investigate and discover nearly all that lay within their originally unconsciously imposed boundaries. "Let it be granted, that a straight line may be drawn from any one point to any other point," "let it be granted, that a circle may be described from any centre, at any distance from that centre," these indicate the limitations of elementary geometry as understood by the Greeks, limitations recognized and formulated in the brightest period of Greek geometry by Euclid. It has been left to later times to recognize that other not less important limitations are implied in the definitions and axioms.

Still striving after the solution of these problems, Archytas, Eudoxus and their successors were led to enlarge their