

It follows that, while with any law of central attraction a circular orbit is possible with any radius r_0 , it will be

$\left\{ \begin{array}{l} \text{stable} \\ \text{unstable} \end{array} \right\}$ according as $\frac{u_0}{P_0} \frac{dP}{du}$ is $\left\{ \begin{array}{l} \text{less} \\ \text{greater} \end{array} \right\}$ than 3.*

8. The case in which $P = \mu u^3$ is peculiar, since the criterion is then identically equal 3. The special case occurs when $C = 0$, the orbit being an equiangular spiral unless $h^2 = \mu$, which makes $\gamma = 90^\circ$, when it becomes a circle, and the circle must be regarded as described with kinetic instability.

LAGRANGE'S PLACE IN THE THEORY OF SUBSTITUTIONS. †

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IN the present brief note I cannot vindicate Lagrange's right to the title of creator of the theory of substitutions; but I hope, by presenting a few examples of his methods, to show the importance of considering him from this point of view. Lagrange was led to the study of this theory by his attempts to solve equations of degree higher than the fourth. Speaking of the inherent difficulties which this thorny subject offered to the investigator, he remarks: ‡

"The theory of equations is of all parts of analysis the one, we would think, which ought to have acquired the greatest degree of perfection, by reason both of its importance and of the rapidity of the progress that its first inventors made; but although the mathematicians of later days have not ceased to apply themselves, there remains much in order that their efforts may meet with the success that one could desire. In regard to the resolution of literal equations one has hardly advanced further than one was in Cardan's time, who was the first to publish the resolution of equations of the third and fourth degree. The first successes of the Italian analysts in this branch seem to have marked the limit of possible discoveries: at least it is certain that all attempts that have been made up to the present to push back the limits of this branch of algebra have hardly served for other purposes than

* An equivalent criterion is otherwise derived in Thomson and Tait's *Natural Philosophy*, § 350.

† Read before the Yale Mathematical Club.

‡ Lagrange: *Nouveaux Mémoires*, Acad. Sciences Berlin, years 1770-71. Also, *Œuvres*, vol. III, pp. 205-421, *Réflexions sur la résolution algébrique des équations*.