

# General properties of the boundary renormalization group flow for supersymmetric systems in $1 + 1$ dimensions

Daniel Friedan<sup>1,2</sup> and Anatoly Konechny<sup>3,4</sup>

<sup>1</sup>Department of Physics and Astronomy,  
Rutgers, The State University of New Jersey,  
Piscataway, NJ 08854-8019, USA

<sup>2</sup>Natural Science Institute, The University of Iceland, Reykjavik, Iceland  
[friedan@physics.rutgers.edu](mailto:friedan@physics.rutgers.edu)

<sup>3</sup>Department of Mathematics, Heriot-Watt University,  
Riccarton, Edinburgh EH14 4AS, UK

<sup>4</sup>Maxwell Institute for Mathematical Sciences, Edinburgh, UK  
[anatolyk@ma.hw.ac.uk](mailto:anatolyk@ma.hw.ac.uk)

## Abstract

We consider the general supersymmetric one-dimensional quantum system with boundary, critical in the bulk but not at the boundary. The renormalization group (RG) flow on the space of boundary conditions is generated by the boundary beta functions  $\beta^a(\lambda)$  for the boundary coupling constants  $\lambda^a$ . We prove a gradient formula  $\partial \ln z / \partial \lambda^a = -g_{ab}^S \beta^b$  where  $z(\lambda)$  is the boundary partition function at given temperature  $T = 1/\beta$ , and  $g_{ab}^S(\lambda)$  is a certain positive-definite metric on the