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ON A CLASS OF L^1-ILLPOSED QUASILINEAR PARABOLIC EQUATIONS

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1. Introduction. The purpose of the present paper is to single out a certain class of quasilinear parabolic equations which admit no solution belonging to L^1 with respect to the space variable.

We consider the equation

$$u_t - \Delta\psi(u) = 0 \qquad \text{in} \qquad \mathcal{R}^N \times (0,T) \tag{1}$$

with $N \geq 2$ and $\psi \in C^{2+\alpha}(\mathcal{R}_+, \mathcal{R})$ satisfying

$$\psi'(u) > 0 \ (u > 0), \quad \lim_{u\to\infty} \psi(u) < \infty, \quad \limsup_{u\to+\infty} \psi'(u) < \infty \tag{2}$$

and

$$-\psi(u) \geq C_0 u^{1-n} \quad (0 < u < \delta), \tag{3}$$

where $\alpha \in (0,1)$, $\delta > 0$, $C_0 > 0$, and $n > 1$ are constants. We interpret (1) in the sense of distributions on $\mathcal{R}^N \times (0,T)$, precisely,

$$\int\int_{\mathcal{R}^N\times(0,T)} (u\eta_t + \psi(u)\Delta\eta)\, dxdt = 0$$

for any $\eta \in C_0^\infty\left(\mathcal{R}^N \times (0,T)\right)$. We study the nonnegative solution $u = u(x,t)$ which belongs to $C\left([0,T), L^1_{loc}\left(\mathcal{R}^N\right)\right)$ and satisfies

$$u_t,\ \psi(u) \in L^1_{loc}\left(\mathcal{R}^N \times (0,T)\right).$$

Following [2] we call it a *strong* solution.

Our first result is stated as follows.

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