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To Professor Włodzimierz Staś on his 75th birthday

DIRICHLET WEAK UNIFORM DISTRIBUTION OF MULTIPLICATIVE FUNCTIONS

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1. Let f be an integer-valued arithmetical function and let $N > 2$ be a given integer. The function f is said to be *weakly uniformly distributed* (mod N) (WUD (mod N)), provided it assumes infinitely many values prime to N and for all i, j prime to N the ratio

$$\frac{\#\{n \leq x : f(n) \equiv i \pmod N\}}{\#\{n \leq x : f(n) \equiv j \pmod N\}}$$

tends to unity, when $x \to \infty$. In the case when f is a polynomial-like multiplicative function (i.e. for $k = 1, 2, \ldots$ one has $f(p^k) = V_k(p)$ for all primes p with suitable $V_k \in Z[X]$) a necessary and sufficient condition for WUD (mod N) has been obtained in [2]. (See also [4] and the literature quoted there). This condition can be restated in a form making sense for multiplicative functions which are not necessarily polynomial-like and in [5] an attempt has been made to reveal its analytical meaning. In that paper the notion of *Dirichlet weak uniform distribution* (mod N) (*Dirichlet-WUD* (mod N)) has been considered and it turned out that for a class of multiplicative functions (encompassing all polynomial-like functions) the said condition is both necessary and sufficient for Dirichlet-WUD (mod N) to hold. However, as noted in the corrigendum to [5] the class of functions to which this condition can be applied is in reality smaller than originally asserted.

The aim of this note is to modify slightly the previous definition of Dirichlet-WUD (mod N) so that it will be applicable to all integer-valued multiplicative functions which assume sufficiently many values prime to N and obtain a necessary

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