

INTEGRAL INVARIANTS OF 3-MANIFOLDS

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Abstract

This note describes an invariant of rational homology 3-spheres in terms of configuration space integrals, which in some sense lies between the invariants of Axelrod and Singer [2] and those of Kontsevich [9].

1. Introduction

In their seminal paper [2] of 1994, Axelrod and Singer showed that the asymptotics of the Chern–Simons theory led to a series of C^∞ -invariants associated to triples $\{M; f; \rho\}$ with M a smooth homology 3-sphere, f a homotopy class of framings of M , and ρ an “acyclic” conjugacy class of orthogonal representations of $\pi_1(M)$. That is, the cohomology $H^*(M; \text{Ad } \rho)$ of M relative to the local system associated to $\text{Ad } \rho$ vanishes.

The primary purpose of this note is to show that the basic ideas of their paper can be adapted quite easily—but not quite trivially—to yield invariants of smooth, framed 3-dimensional homology spheres as such. Put differently, we will present a treatment somewhat analogous to theirs for the trivial representation of $\pi_1(M)$. We say somewhat because in our work we have put aside the physics inspired aspects of Axelrod and Singer’s paper. Instead we have simply taken our task to be the production of invariants of framed manifolds $(M; f)$ out of some fixed Riemannian structure on M .

There is of course Kontsevich’s solution by “softer methods” to the problem of finding the residual invariants of the Chern–Simons theory

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