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# EQUIVARIANT HOLOMORPHIC MORSE INEQUALITIES I: A HEAT KERNEL PROOF

VARGHESE MATHAI & SIYE WU

## Abstract

Assume that the circle group acts holomorphically on a compact Kähler manifold with isolated fixed points and that the action can be lifted holomorphically to a holomorphic vector bundle. We use some techniques developed by Bismut and Lebeau to give a heat kernel proof of the equivariant holomorphic Morse inequalities, which, first obtained by Witten using a different argument, produce bounds on the multiplicities of weights occurring in the twisted Dolbeault cohomologies in terms of the data of the fixed points.

## 1. Introduction

Morse theory gives some topological information of manifolds by means of the critical points of functions. Let $h$ be a Morse function on a compact manifold of real dimension $n$ and suppose that $h$ has isolated critical points only. Let $m_k$ $(0 \leq k \leq n)$ be the $k$-th Morse number, the number of critical points of Morse index $k$. The Hopf formula for the gradient vector field says that the alternating sum of $m_k$ is equal to that of the Betti numbers $b_k$:

$$\sum_{k=0}^{n}(-1)^k m_k = \sum_{k=0}^{n}(-1)^k b_k. \tag{1.1}$$

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