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EINSTEIN TYPE METRICS AND STABILITY ON VECTOR BUNDLES

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Abstract

In this paper we show that stability for holomorphic vector bundles are equivalent to the existence of solutions to certain system of Monge Ampère equations parametrized by a parameter k. We solve this fully nonlinear elliptic system by singular perturbation technique and show that the vanishing of obstructions for the perturbation is given precisely by the stability condition. This can be interpreted as an infinite dimensional analog of the equivalency between Geometric Invariant Theory and Symplectic Reduction for moduli space of vector bundles.

1. Introduction

This paper is largely grown out from the thesis [7] of the author under the direction of Professor S.T.Yau. However, a more concrete picture in terms of the infinite dimensional Geometric Invariant Theory (GIT) and the symplectic reduction will also be presented here.

We shall demonstrate that when we use GIT to study the moduli problem of vector bundles (following Gieseker), it is equivalent to finding certain canonical Einstein type metrics on the bundle E. The curvature of such metrics satisfies a fully nonlinear elliptic system of equations arised as moment map equations (the almost Hermitian Einstein equations):

$$[e^{\frac{i}{2\pi}R_A+k\omega I_E}Td(X)]^{(2n)} = \frac{1}{rk(E)}\chi(X, E\otimes L^k)\frac{\omega^n}{n!}I_E.$$

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