

HOLOMORPHIC MAPS OF A RIEMANN SURFACE INTO A FLAG MANIFOLD

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Abstract

We prove that there exist constants $c_0 > 0, c_1$ such that the inclusion

$$\text{Hol}_k^0(\Sigma, G/P) \rightarrow \text{Map}_k^0(\Sigma, G/P)$$

of the space of holomorphic maps of degree $k = (k_1, k_2, \dots, k_r), k_i \geq 0$ of a Riemann surface Σ to a flag manifold G/P , into the corresponding space of continuous maps, induces isomorphisms in homology groups H_i for $i < c_0|k| - c_1$, with $|k| = \min(k_i)$.

1. Introduction

The space of based smooth maps $\text{Map}^0(\Sigma, X)$ from a compact surface Σ into a manifold X can be considered from many points of view, in terms of its topology, its geometry, and, thanks to string theory, even its physics. When both Σ and X are Riemannian, there is a natural energy functional

$$(1.1) \quad E(f) = \int_{\Sigma} |df|^2$$

defined on $\text{Map}^0(\Sigma, X)$, whose critical points are harmonic maps. One class of manifolds X that has been considered quite extensively is that of the generalised complex flag manifolds G/P , G a semi-simple complex Lie group, and P a parabolic subgroup. In this case, the minima of the functional correspond to holomorphic or anti-holomorphic maps, and in general, there are other, non-minimal, critical points.

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