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REMARKS ON COMPLETE DEFORMABLE HYPERSURFACES IN R^4

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Dedicated to Professor T. Otsuki on his 75th birthday and to Professor S. Ishihara on his 70th birthday

Abstract

It is shown that, for each pair $\{k_1(u), k_2(v)\}$ of smooth functions on R with some conditions, there exists a family of complete nonruled deformable hypersurfaces $M(\lambda, k_1, k_2)$, $-\frac{1}{2} < \lambda < \frac{1}{2}$, in Euclidean space R^4 with rank $\rho = 2$ almost everywhere. This is an answer to one of the problems in [3].

1. Introduction and statement of results

It is an interesting problem to determine the deformability of an isometric immersion f of a connected Riemannian manifold M^n into Euclidean $(n+1)$-space R^{n+1}, $n \geq 3$. Let ρ be the rank of the second fundamental form of f. It is known (see [2]) that f is rigid (i.e., not deformable) if $\rho \geq 3$ by the Beez–Killing Theorem, and highly deformable if $\rho \leq 1$. The situation for constant rank $\rho = 2$ is quite complicated. Sbrana and Cartan divided this situation into three different types, and looked into it by a detailed local analysis (see [1], [4]).

It has been shown by Dajczer and Gromoll [3] that for $n \geq 3$ a complete hypersurface M^n in R^{n+1} whose set of all the geodesic points does not disconnect M^n, is rigid unless it contains either an open subset $L^3 \times R^{n-3}$ with L^3 unbounded or a complete ruled strip. But the three-dimensional case of this result remains an open problem.

In this paper, we construct a one-parameter family of complete nonruled deformable hypersurfaces in R^4 with rank $\rho = 2$ almost everywhere depending on two functions on the real line R with some conditions.

Theorem. *Let $k_j(x)$, $j = 1, 2$, be smooth functions on R satisfying that $-\frac{\pi}{4} < \int_0^x k_j(x)\,dx < \frac{\pi}{4}$, $j = 1, 2$, $\forall x \in R$ and that $k_1(u) > 0$, $k_2(v) < 0$ at all points u, v except for isolated ones. For each constant*

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