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THE EXISTENCE OF ANTI-SELF-DUAL CONFORMAL STRUCTURES

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1. Introduction

The following is a salient feature of 4-dimensional Riemannian geometry: The conformal class of a Riemannian metric on an oriented 4-manifold defines a splitting

$$\Lambda^2 T^* \simeq \Lambda^2_+ T^* \oplus \Lambda^2_- T^*. \tag{1.1}$$

The bundles $\Lambda^2_\pm T^*$ are real, 3-plane bundles whose sections are called self-dual $(+)$ or anti-self-dual $(-)$ 2-forms.

With the metric's help, the Riemannian curvature can be thought of as a section of the symmetric endomorphisms of $\Lambda^2 T^*$. Then, with respect to (1.1), this section, $\mathscr{R}$, has the form

$$\mathscr{R} = \left(\begin{array}{c|c} \mathscr{W}_+ + \frac{s}{12}\cdot 1 & B \\ \hline B^T & \mathscr{W}_- + \frac{2}{12}\cdot 1 \end{array}\right). \tag{1.2}$$

Here, s is the usual scalar curvature, B is the traceless Ricci tensor (in an unusual guise), and the $\mathscr{W}_\pm$ are, respectively, the self-dual and anti-self-dual Weyl tensors. (The metric is Einstein if $B = 0$, and it is conformally flat if $\mathscr{W}_+$ and $\mathscr{W}_-$ are both zero.)

(a) **Existence.** Given that this preamble is understood (and [1] is the canonical reference), it can be said that the purpose of this article is to discuss metrics with $\mathscr{W}_+ = 0$. We give the main result:

Theorem 1.1. *Let M be a smooth, compact, oriented, 4-dimensional manifold. Use $\underline{\mathbb{CP}}^2$ to denote complex projective 2-space with the opposite of its complex orientation. Use # to denote the operation of connect sum. For all sufficiently large N, $M_N \equiv M\#_N\underline{\mathbb{CP}}^2$ admits a metric with $\mathscr{W}_+ \equiv 0$.*

Remark that the connect sum of manifolds X and Y is obtained from their disjoint union by cutting out an open ball in X and one in Y and then identifying the two resulting boundary 3-spheres.

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