J. DIFFERENTIAL GEOMETRY
33 (1991) 551-573

SELF-DUAL CONFORMAL STRUCTURES ON $l\mathbb{C}P^2$

ANDREAS FLOER

Abstract

We prove the existence of conformal structures with self-dual Weyl tensor on connected sums of arbitrarily many copies of two-dimensional complex projective space $\mathbb{C}P^2$. They are constructed from the standard conformal structures on $\mathbb{C}P^2$ by a gluing procedure.

1. Introduction

A conformal structure c on a smooth finite dimensional manifold M is an equivalence class $c = [g]$ of Riemannian metrics g on M, where $g_1 \sim g_2$ are (conformally) equivalent if $g_2 = f \cdot g_1$ for a smooth function $f: M \to \mathbb{R}_+$, the set of positive real numbers. We say that $(M, [g])$ is conformally flat if there exists a system of charts ψ, $M \supset U \to \mathbb{R}^n$ such that $\psi^* g \sim g_0$, where $\mathbb{R}^n$ is a Euclidean n-space, and g_0 is the Euclidean metric. The condition for the existence of such a restricted atlas can be stated as a nonlinear partial differential equation, called the integrability condition, on the conformal structure itself.

In two dimensions, a conformal structure is precisely specified by assigning an orthogonal direction to each direction in the tangent space $T_x M$ of M at $x \in M$. If M is orientable, this yields a 1-1 correspondence with complex structures on M, so that a diffeomorphism ϕ of M is conformal (i.e., $\phi^* g \sim g$) if and only if it is holomorphic. It follows that every orientable conformal 2-dimensional manifold allows a conformal atlas, since it allows a holomorphic one. In dimensions higher than two, the set of conformal diffeomorphisms is much smaller. For the constant conformal structure on $\mathbb{R}^n$, $n > 2$, for example, it is a finite-dimensional group. Correspondingly, it is less likely to find a conformal atlas of M. In dimensions higher than three, the integrability condition for conformal structures is the Weyl tensor W, which is a component of the Riemannian curvature tensor R, i.e., of the integrability condition for the metric itself. (In dimension 3, the integrability condition is a first order differential

Received by the editors October 1, 1987 and, in revised form, October 30, 1989.