J. DIFFERENTIAL GEOMETRY
29 (1989) 85-94

# A CONSTRUCTION OF METRICS OF NEGATIVE RICCI CURVATURE

ROBERT BROOKS

In this paper we will prove

**Theorem 1.** *Every compact 3-manifold admits a metric of negative Ricci curvature.*

This theorem was originally proved by Gao and Yau in [2]. The proof we give here, while based on [2], is substantially shorter, given signficant results about hyperbolic 3-manifolds, and is (to our taste) more constructive and conceptual. The simplicity comes from staying as long as possible in the category of hyperbolic metrics. In particular, we make strong use of the existence of tubular neighborhoods of specified width about short geodesics. This theory follows from Jorgensen's inequality [4] and was developed in [1] and [7].

Our construction is flexible enough to give

**Theorem 2.** *There exist positive constants $a$ and $b$ such that every compact 3-manifold admits a metric whose Ricci curvatures all lie between $-a$ and $-b$.*

Observe that if $M$ admits a metric whose Ricci curvatures lie between $-a$ and $-b$, where $b/a < 2$, then this metric has negative sectional curvature. Our argument gives a ratio of $b/a$ on the order of 1,000, but we do not compute it explicitly.

We also obtain results about higher-dimensional manifolds carrying metrics of negative Ricci curvature:

**Theorem 3.** *Let $M$ be a hyperbolic orbifold of order $k$, where $k \geq 12$. Then $M$ admits a metric of negative Ricci curvature.*

The terminology "hyperbolic orbifold of order $k$" is explained in §4. The proof of Theorem 3 was motivated by the paper [3], which has a number of points of contact with the present paper.

The plan of our argument is as follows.

Given a 3-manifold $M$, it follows from the Thurston theory [8] that there is a link $L$ in $M$ such that $M - L$ has a complete metric of constant curvature

Received July 2, 1987. The author was partially supported by National Science Foundation Grant DMS-8501300.